Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

sam@nitk.edu.in https://sam.nitk.ac.in/

MA111 - Engineering Mathematics - II
Problem Sheet - 10

Higher Order Linear ODE

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1. Find the Differential equation with the following solution $y = ae^x + be^{-x} + ccosx + dsinx$ where a,b,c,d are parameters (Ans:$y^{iv} - y = 0$)

2. Show that $v = B + \frac{A}{r}$ is a solution of $\frac{d^2v}{dr^2} + \frac{2}{r} = 0$.

3. Show that
   (a) $sinx, sin2x, sin3x$ are Linearly Independent on [0,2π]
   (b) $e^x, sinx, cosx$ are Linearly Independent on [0,2π]
   (c) $sinx, sin(x + \pi/8), sin(x - \pi/8), (-\infty, \infty)$ are Linear Dependent.
   (d) $1, x, x^2, ...x^n, (-\infty, \infty)$ are Linearly Independent

4. Are the given functions linearly independent or dependent on the positive x-axis?
   (a) $1, e^x, e^{-x}$ (Ans:Linearly Independent)
   (b) $lnx, lxx^2, (lnx)^2$ (Ans:Linearly dependent)
   (c) $x^2, x|x|, x$ (Ans: If x¿0,Linearly Dependent)
   (d) $sinx, sin2x, cosx$ (Ans:Linearly Independent)
   (e) $sinx, sin\frac{1}{2}x$ (Ans:Linearly Independent)
   (f) $cos^2x, sin^2x, 2\pi$ (Ans:linearly Dependent)

5. Solve the Differential Equation
   (a) $y''' + 6y'' + 11y' + 6y = 0(Ans : y = c_1e^{-x} + c_2e^{-2x} + c_{e^{-3x}})$
   (b) $y^{iv} + 2y''' - 3y'' - 4y' + 4y = 0$ (Ans: $(c_1 + c_2x)e^{-x} + (c_3 + c_4x)e^{-2x}$)
   (c) $y''' - 8y = 0(Ans : c_1e^{2x} + e^{-x}(c_2cos(x\sqrt{3}) + c_3sin(x\sqrt{3}))$
   (d) $(y'' - 2y' + 5y)^2 = 0$ (Ans:$e^x((c_1 + c_2x)cos2x + (c_3 + c_4x)sin2x)$
   (e) $y''' + y''(2\sqrt{3} - 1) + y'(3 - 2\sqrt{3}) - 3y = 0$ (Ans: $y = c_1e^x + (c_2 + c_3x)e^{-x\sqrt{3}}$)
   (f) $y^v - 13y''' + 26y''82y' + 104y = 0$ (Ans: $y = c_1e^{-4x} + e^{-x}(c_2cosx + c_3sinx) + e^{3x}(c_4cos2x + c_5sin2x)$)
   (g) $y''' + 3.2y'' + 4.81y' = 0.y(0) = 3.4, y'(0) = -4.6, y''(0) = 9.91$ (Ans: $y = 2.4 + e^{-1.6x}(cos1.5x - 2sin1.5x)$)
   (h) $y^{iv} - 9y'' - 400y = 0, y(0) = 0, y'(0) = 0, y''(0) = 41, y'''(0) = 0$ (Ans:$y = cosh5x - cos4x$)

6. Using Unedetrmined Co-efficient, Solve

(a) $y''' + 3y'' + 2y' = x^2 + 4x + 8$ (Ans: $y = c_1 + c_2e^{-x} + c_3e^{-2x} + \frac{x}{12}(33 + 3x + 2x^2)$)

(b) $y''' + y' = 2x^2 + 4sinx$ (Ans:$y = c_1 + c_2cosx + c_3sinx - 4x + \frac{2}{3}x^3 - 2xsinx$)

(c) $y''' - 7y' - 6y = e^{-2x}$ (Ans: $c_1e^{-x} + c_2e^{3x} + c_3e^{-2x} + \frac{x}{5}e^{-2x}$)

(d) $y''' - y'' - 4y' = 4y = 2x^2 - 4x - 1 + 2x^2e^x + 5xe^{2x} + e^{2x}$ (Ans: $y = c_1e^x + c_2e^{2x} + c_3e^{-2x} + \frac{1}{2}x^2 + \frac{1}{6}x^3e^{2x}$

(e) $y''' - 3y'' + 2y' = x^2e^x$ (Ans:$y + c_1 + c_2e^x + c_3e^{2x} - 2xe^x - \frac{x^3}{3}e^x$)

(f) $y''' - 3y'' + 2y' = 3xe^x + 5x^3$ (Ans:$y = c_1 + c_2e^x + c_3e^{2x} - (\frac{9}{4})xe^{2x} + (5x^4 + 30x^3 + 105x^2 + 225x)/8$

(g) $y''' + y'' = cos2x, y(0) = 1, y'(0) = 2, y''(0) = 3.$ (Ans:$y = -3.6 + 4.7x + 2.6e^{-x} - 0.1cos2x - 0.05sin2x$)

(h) $y''' - 9y'' + 27y' - 27y = 54sin3x, y(0) = 3.5, y'(0) = 13.5, y''(0) = 38.5$ (Ans: $y = (4 - x^2)e^{3x} - 0.5cos3x + 0.5sin3x$)

7. Solve the following equations by the Method of varaition of parameter

(a) $y''' + 4y'' = 4cotx$ (Ans:$y = c_1 + c_2cos2x + c_3sin2x + (\frac{1}{2})log(sin2x) - \frac{1}{2} + (\frac{1}{2})cos2xlog(tanx)$)

(b) $x^3y''' + x^2y'' - 2xy' + 2y = xlogx$ (Ans:$y = c_1x + c_2x^{-1} + c_3x^2 - (\frac{x}{4})((logx)^2 + logx) - (\frac{3x}{8})$)

(c) $y'' + 2y' + 2y = 4e^{-x}sec^3x$ (Ans:$y = e^{-x}(c_1cosx + c -_2 sinx + 2sinxtanx)$)

(d) $y''' - 6y'' - 11y' - 6y = e^{2x}$ (Ans: $y = c_1e^x + c_2e^{2x} + c_3e^{3x} - xe^{2x}$)

(e) $y''' + y' = secx$ (Ans: $c_1 + c_2cosx + c_3sinx + log(secx + tanx) - xcosx + sinxlog(cosx)$)

(f) $y''' + y' = cosecx$ (Ans: $c_1 + c_2cosx + c_3sinx + log(cosecx + cotx) - xsinx - cosxlog(sinx)$)

(g) $y'' - 2y' = \frac{e^x}{4x}$ (Ans: $y = c_1e^x + c_2xe^x + (\frac{(-e^xx) + (e^xxlnx)}{4})$)

(h) $x^3y''' - 4x^2y'' + 8xy' + 8y = 6x^3(x^2 + 1)^{\frac{-3}{2}}$ (Ans:$y = c_1x + c_2x^2 + c_3x^4 - 2x(x^2 + 1)^{(\frac{-3}{2})}$)

(i) $x^3y''' - x^2y'' - 7xy' + 16y = 9xlnx, y(1) = 6, y'(1) = 18, y''(10 = 65)$ (Ans:$y = x^{-2} - x^2 + 5x^4 + x(lnx + 1)$)

(j) $y''' + 4y'' + 85y' = 135e^x, y(0) = 10.4, y'(0) = -18.1, y''(0) = -691.6$ (Ans:$3 + 9e^{-2x}cos9x - (1.6 - 1.5x)e^x$))

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